

## **1800 MHz Auction**

**Award of frequencies in the frequency bands  
1720.1-1785.0 MHz and 1815.1-1880.0 MHz**

## **Examples of bid evaluation in the third auction stage**

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## 1 Introduction

This note provides some worked examples of the bid evaluation process in the third auction stage of the Danish 1800 MHz Auction. This covers:

- the assessment of whether the third auction stage can end (and the determination of winning bids); and
- the determination of lot categories for which round prices will increase if a further round is needed.

For a description of the process, please refer to the Information Memorandum.

In most of the examples we assume that there are three bidders. We start with the simplest case in which there are no exemption lots. We then introduce examples for the case where a single exemption lot category is included in the third auction stage, followed by examples for the case in which all exemption lot categories are included in the third auction stage. Finally, we provide some examples for cases with a different number of bidders (two and four bidders).

## 2 Examples of bid evaluation when no exemption lot categories are included in the third auction stage

The first few examples cover the simple case in which no exemption lots are available. This will happen if the three available A lots are assigned in the first auction stage. In this case, only seven B lots, which are not subject to any coverage obligation, will be offered in the third auction stage.

The bid evaluation process in this case is simple. As bids cannot include C lots, we only need to consider the single scenario in which there are only B lots available. Equally, if a further round is needed, then the price of B lots must increase.

In the following examples for the case in which there are no exemption lots available we only consider the scenario in which the supply of exemption lots is zero, and skip the process for determining which lot categories require a price increment when a further round is needed. However, it is possible to use the

complete bid evaluation process if no exemption lots are included in the third auction stage, which will lead to the same results.<sup>1</sup>

In the first two examples we provide a detailed description of the individual steps; for the remaining examples we only provide a short commentary on the example followed by the list of bids to be evaluated and the results of the evaluation (presented in tabular form). Each bid is identified with an id, which starts with “H” if the bid is a headline bid in the most recent round and with “A” otherwise. The bid is then followed by a number identifying the bidder who submitted the bid, and if there are multiple additional bids a further “.” and a second number to distinguish the different additional bids submitted by the bidder.

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<sup>1</sup> When following the complete process we go through each lot category in turn to create the hypothetical bids based on the bidder’s headline bid, where we retain the demand for the lot category under consideration and set the demand for any other lots to zero. If there is only one lot category, the hypothetical bid when considering this lot category is identical to the headline bid. Therefore, when we re-process bids using this bid we obtain the same results and thus a bidder who is an omitted bidder with its actual bids continues to be an omitted bidder when using the hypothetical bid instead of its headline bid. This means that we identify the only lot category as requiring a price increment.

## Example 1

There are three bidders, and we consider the evaluation of bids in a round in which the round price for B lots has reached DKK 30 million.

Suppose that up to this round, all three bidders have made headline bids for four B lots, and no additional bids. We therefore only need to consider the highest bid submitted by each bidder for its respective package, i.e. the following three bids:

Bid id	Bidder	Headline/additional	B lots	Amount (DKK million)
H1	Bidder1	Headline	4	120
H2	Bidder2	Headline	4	120
H3	Bidder3	Headline	4	120

### Step 1

#### **Feasible bid combinations:**

As there are only seven B lots, it is not feasible to accept more than one bid simultaneously. Therefore, there are only four feasible bid combinations, namely one each for accepting the bid from one of the bidders, and one in which no bids are accepted.

#### **Highest-value feasible bid combinations:**

The value of not accepting any bids is the reserve price of the seven lots, i.e. DKK 175 million.

The value of accepting a single bid is the bid amount (which for any of the three bids considered is DKK 120 million) plus the reserve price of lots that would remain unassigned (which for any of these combinations would be DKK 75 million, as three lots would remain unassigned). Therefore the total value of any of the three combinations involving a single bid is DKK 195 million.

Accepting an individual bid has greater value than not accepting bids. Therefore, the highest-value feasible bid combinations are the three combinations that involve accepting a single bid.

#### **Exemption-compatible highest-value feasible bid combinations:**

All feasible bid combinations are exemption-compatible in the scenario without exemptions.

## Step 2

### ***Possible winning bid combinations:***

The possible winning bid combinations are the exemption-compatible, highest-value feasible bid combinations that achieve the highest value across all scenarios. As there is only one scenario to be considered, the three exemption-compatible highest-value feasible bid combinations are also the possible winning bid combinations.

## Step 3

### ***Check if any of the possible winning bid combinations includes a bid from each bidder:***

Each of the possible winning bid combinations includes a single bid. Therefore, none of the combinations includes a bid from each bidder.

A further round is needed, and the price of B lots will need to increase.

## Example 2

Building on Example 1, suppose that the price of B lots for the following round is increased to DKK 31 million. Suppose that at this point all three bidders reduce their demand: Bidder 1 switches to a package containing three B lots (with a bid amount of DKK 93 million), whilst bidders 2 and 3 contract their demand further by switching to a package containing two B lots (with a bid amount of DKK 62 million).

We only need to consider the highest bid submitted by each bidder for each package. However, we now have the bids for four lots submitted in the previous round, and the bids submitted in the present round. Therefore we need to consider the following bids (headline bids from the previous round are treated as additional bids in the current round):

Bid id	Bidder	Headline/additional	B lots	Amount (DKK million)
H1	Bidder1	Headline	3	93
A1.1	Bidder1	Additional	4	120
H2	Bidder2	Headline	2	62
A2.1	Bidder2	Additional	4	120
H3	Bidder3	Headline	2	62
A3.1	Bidder3	Additional	4	120

### Step 1

#### ***Feasible bid combinations:***

The number of feasible bid combinations increases substantially once we introduce bids for fewer lots. Aside from the bid combinations in the previous round (i.e. accepting each of the headline bids from the previous round individually, or not accepting any bids) we now have the possibility of accepting the headline bids from the current round individually; in pairs (combined with other headline bids from this round or from the previous round); and also the possibility of accepting the three headline bids submitted this round.

#### ***Highest-value feasible bid combinations:***

The value of a bid combination is the sum of bid amounts in the combination plus the reserve price of lots that would remain unassigned. The highest value can be achieved when accepting the headline bids from this round (H1, H2 and H3), with a total value corresponding to the amount of the three bids, i.e. DKK 217 million; no lots remain unassigned. All other feasible combinations have a lower value, as they involve a bid made at the lower round prices of the previous round or including some unassigned lots at the even lower reserve price. Therefore the combination (H1, H2 and H3) is the single highest-value feasible bid combination.

#### ***Exemption-compatible highest-value feasible bid combinations:***

All feasible bid combinations are exemption-compatible in the scenario without exemptions.

## Step 2

### ***Possible winning bid combinations:***

There is only one possible winning bid combination, the combination (H1, H2 and H3).

## Step 3

### ***Check if any of the possible winning bid combinations includes a bid from each bidder:***

The combination (H1, H2 and H3) includes a bid from each bidder.

The third auction stage ends.

### Example 3

This is a slight variation of Example 2. Following the situation in Example 1, the price is increased to DKK 31 million. The three bidders make bids for the same packages as in Example 2, but the bids amounts differ. Specifically:

- Bidder 2 contracts its demand to two lots without increasing its bid for four lots, as in Example 2;
- Bidder 1 continues to make a headline bid for four B lots as in the preceding round, and submits an additional bid for a package of three lots with a bid amount that is below the round price for the package; and
- Bidder 3 also continues to make headline bids for four B lots as in the preceding round, and submits an additional bid for a package of two lots with a bid amount that is below the round price for the package.

As a result, accepting bids from all three bidders no longer yields the highest value (and specifically accepting the headline bid from Bidder 2 along with the headline bid from Bidder 1 or Bidder 3 yields a higher value even if it results in one lot remaining unassigned). Accordingly, a further round is needed and the price of B lots needs to increase.

Bids (round price for B lots is DKK 31 million)

Bid id	Bidder	Headline/additional	B lots	Amount (DKK million)
H1	Bidder1	Headline	4	124
A1.1	Bidder1	Additional	3	80
H2	Bidder2	Headline	2	62
A2.1	Bidder2	Additional	4	120
H3	Bidder3	Headline	4	124
A3.1	Bidder3	Additional	2	60

Bid evaluation

Highest value (DKK million)	Highest-value feasible bid combinations	Exemption-compatible?	Possible winning bid combination?	Includes a bid from each bidder?
211	(H1, H2)	Yes	Yes	No
	(H2, H3)	Yes	Yes	No



## Example 4

In this example we show a situation in which it would be feasible to accept the headline bids from all three bidders given the lots available. However, this would leave many lots unassigned and thus does not yield the highest value. A higher value is achieved when accepting additional bids from only two bidders.

As a result, a further round is required and the price of B lots needs to increase.

Bids (round price for B lots is DKK 31 million)

Bid id	Bidder	Headline/additional	B lots	Amount (DKK million)
H1	Bidder1	Headline	1	31
A1.1	Bidder1	Additional	4	120
H2	Bidder2	Headline	1	31
A2.1	Bidder2	Additional	3	90
A2.2	Bidder2	Additional	4	120
H3	Bidder3	Headline	1	31
A3.1	Bidder3	Additional	4	110

## Bid evaluation

Highest value (DKK million)	Highest-value feasible bid combinations	Exemption-compatible?	Possible winning bid combination?	Includes a bid from each bidder?
210	(A1.1, A2.1)	Yes	Yes	No

## Example 5

This example is similar to Example 4 in that it would be feasible to accept the headline bids from all three bidders given the lots available, but this again does not yield the highest value. However, in this example it is possible to achieve the highest value with a combination that includes one bid from each bidder. Therefore, the third auction stage can end.

Bids (round price for B lots is DKK 31 million)

Bid id	Bidder	Headline/additional	B lots	Amount (DKK million)
H1	Bidder1	Headline	1	31
A1.1	Bidder1	Additional	4	120
A1.2	Bidder1	Additional	2	60
H2	Bidder2	Headline	1	31
A2.1	Bidder2	Additional	3	90
A2.2	Bidder2	Additional	4	120
H3	Bidder3	Headline	1	31
A3.1	Bidder3	Additional	4	110

## Bid evaluation

Highest value (DKK million)	Highest-value feasible bid combinations	Exemption-compatible?	Possible winning bid combination?	Includes a bid from each bidder?
211	(A1.2, A2.2, H3)	Yes	Yes	Yes

## Example 6

This example is similar to Example 5, but in this case one of the bidders submits the zero bid as its headline bid. Despite this, the bidder who had submitted the zero bid has one of its additional bids selected as a winning bid and wins two B lots.

Bids (round price for B lots is DKK 31 million)

Bid id	Bidder	Headline/additional	B lots	Amount (DKK million)
H1	Bidder1	Headline	0	0
A1.1	Bidder1	Additional	4	120
A1.2	Bidder1	Additional	2	60
H2	Bidder2	Headline	1	31
A2.1	Bidder2	Additional	3	90
A2.2	Bidder2	Additional	4	120
H3	Bidder3	Headline	1	31
A3.1	Bidder3	Additional	4	110

## Bid evaluation

Highest value (DKK million)	Highest-value feasible bid combinations	Exemption-compatible?	Possible winning bid combination?	Includes a bid from each bidder?
211	(A1.2, A2.2, H3)	Yes	Yes	Yes

## Example 7

With a slight modification of the previous example we obtain an outcome in which the bidder who has submitted the zero bid as its headline bid does not win any lots. The difference to the previous example is that now Bidder 1 does not submit an additional bid for two lots (which was its winning bid in Examples 5 and 6) and that Bidder 3's headline bid is for three B lots.

Note that in this case there are three possible winning bid combinations. However, only one of these combinations includes a bid from each bidder, and thus can be the winning combination.

Bids (round price for B lots is DKK 31 million)

Bid id	Bidder	Headline/additional	B lots	Amount (DKK million)
H1	Bidder1	Headline	0	0
A1.1	Bidder1	Additional	4	120
H2	Bidder2	Headline	1	31
A2.1	Bidder2	Additional	3	90
A2.2	Bidder2	Additional	4	120
H3	Bidder3	Headline	3	93
A3.1	Bidder3	Additional	4	110

## Bid evaluation

Highest value (DKK million)	Highest-value feasible bid combinations	Exemption-compatible?	Possible winning bid combination?	Includes a bid from each bidder?
213	(H1, A2.2, H3)	Yes	Yes	Yes
	(A2.2, H3)	Yes	Yes	No
	(A1.1, H3)	Yes	Yes	No

## Example 8

In this example we find that a further round is needed despite the fact that only one bidder is making a headline bid that includes B lots, whilst the other bidders make zero bids. Despite this, the highest value can be obtained by accepting additional bids from the two bidders who are submitting a zero headline bid. Therefore, a further round is needed and the price of B lots needs to increase to allow Bidder 3 to increase its bid to outbid its competitors if it wishes to do so.

Bids (round price for B lots is DKK 31 million)

Bid id	Bidder	Headline/additional	B lots	Amount (DKK million)
H1	Bidder1	Headline	0	0
A1.1	Bidder1	Additional	3	90
A1.2	Bidder1	Additional	4	120
H2	Bidder2	Headline	0	0
A2.1	Bidder2	Additional	3	90
A2.2	Bidder2	Additional	4	120
H3	Bidder3	Headline	2	62
A3.1	Bidder3	Additional	4	110

## Bid evaluation

Highest value (DKK million)	Highest-value feasible bid combinations	Exemption-compatible?	Possible winning bid combination?	Includes a bid from each bidder?
210	(A1.1, A2.2) (A1.2, A2.1)	Yes Yes	Yes Yes	No No

## Example 9

In our final example without exemption lots we show a situation in which all bidders submit zero headline bids, along with some additional bids. There are three possible winning outcomes that include a bid from each bidder, but only one of them includes a non-zero bid from each bidder, which would be selected in accordance with the tie-breaking criteria.

Bids (round price for B lots is DKK 31 million)

Bid id	Bidder	Headline/additional	B lots	Amount (DKK million)
H1	Bidder1	Headline	0	0
A1.1	Bidder1	Additional	4	120
A1.2	Bidder1	Additional	2	60
H2	Bidder2	Headline	0	0
A2.1	Bidder2	Additional	3	90
A2.2	Bidder2	Additional	2	60
H3	Bidder3	Headline	0	0
A3.1	Bidder3	Additional	3	90

### Bid evaluation

Highest value (DKK million)	Highest-value feasible bid combinations	Exemption-compatible?	Possible winning bid combination?	Includes a bid from each bidder?
210	(A1.1, A2.1)	Yes	Yes	No
	(A1.1, A2.1, H3)	Yes	Yes	Yes
	(A1.1, A3.1)	Yes	Yes	No
	(A1.1, H2, A3.1)	Yes	Yes	Yes
	(A1.2, A2.2, A3.1)	Yes	Yes	Yes

The combination (A1.2, A2.2, A3.1) is selected as the winning combination pursuant to the second tie-breaking criterion (select outcome with greater number of non-zero bids).

## 3 Examples with a single exemption lot category

We now present some examples for the case where one exemption lot category is included in the third auction stage. This will happen if two of the three available A lots are assigned in the first auction stage. In this case, nine B lots will be offered in the third auction stage, and one exemption lot category (exemptions from the coverage obligation in the coverage area group that has not been assigned in the second auction stage) is included. In the following examples we assume that Bidder 3 has not been assigned an A lot and that category C1 is available.

The bid evaluation process in this case requires assessing all the scenarios in relation to the number of exemption lots available. Below  $S_i$  denotes the scenario in which there are  $i$  exemption lots available in each category included in the third auction stage.

As there are now two lot categories, establishing which of these need a price increment requires the process set out in the Information Memorandum.

Again, we provide a more detailed description of the individual steps in the first two examples, and provide a short commentary about the example followed by the list of bids to be evaluated and the results of the evaluation for the remaining examples. The results of the evaluation are presented in two tables, one for the assessment of the closing condition, the other one for the assessment of price increments.

## Example 10

There are three bidders. We consider the evaluation of bids after the first round, when the round price for B lots is DKK 25 million and the price for C1 lots is zero. Suppose that the three bidders make only headline bids; Bidder 1 and Bidder 2 bid for four B lots and C1, and Bidder 3 bids for six B lots and C1. We need to consider the following three bids:

Bid id	Bidder	Headline/additional	B lots	C1	Amount (DKK million)
H1	Bidder1	Headline	4	1	100
H2	Bidder2	Headline	4	1	100
H3	Bidder3	Headline	6	1	150

### Step 1

#### **Feasible bid combinations:**

As there are three bidders, we need to consider three hypothetical scenarios:

- $S_0$ : no C1 lots available;
- $S_1$ : 1 C1 lot available; and
- $S_2$ : 2 C1 lots available.

$S_0$ : If no C1 lots are available it is not possible to accept any bids in this scenario as all of the bids include one C1 lot. Therefore, the only feasible combination is to not accept any bids.

$S_1$ : If one C1 lot is available we can only accept one bid or no bids at all as all of the bids include one C1 lot.

$S_2$ : If two C1 lots are available we can consider accepting pairs of bids in addition to the combinations that were feasible in the previous two scenarios. It is possible to accept H1 and H2 simultaneously. However, it is not possible to accept H3 along with another bid, as we only have nine B lots.

#### **Highest-value feasible bid combinations:**

As round prices are equal to reserve, all of these combinations have the same value, equal to the reserve price of all lots, i.e. DKK 225 million. Therefore, all feasible combinations in each scenario are highest-value feasible bid combinations for that scenario.

#### **Exemption-compatible highest-value feasible bid combinations:**

All feasible bid combinations are exemption-compatible in the scenario without exemptions.

For all other scenarios, a feasible bid combination is exemption compatible if it includes at least  $n+1$  non-zero bids, where  $n$  is the number of exemption lots available in that scenario. Therefore:

- a feasible bid combination in  $S_1$  is only exemption-compatible if it includes at least two non-zero bids; and



- a feasible bid combination in  $S_2$  is only exemption-compatible if it includes at least three non-zero bids.

Each of the feasible combinations in  $S_1$  includes at most one non-zero bid. Therefore, none of the highest-value feasible bid combinations in this scenario are exemption-compatible.

Each of the feasible combinations in  $S_2$  includes at most two non-zero bids. Therefore, none of the highest-value feasible bid combinations in this scenario are exemption-compatible.

## Step 2

### **Possible winning bid combinations:**

The possible winning bid combinations are the exemption-compatible, highest-value feasible bid combinations that achieve the highest value across all scenarios. In this example we only consider further the highest-value feasible bid combination from  $S_0$  (the scenario with no exemptions), which includes no bids. Therefore, there is one single possible winning bid combination, which is not to accept any bids.

## Step 3

### **Check if any of the possible winning bid combinations includes a bid from each bidder:**

The single possible winning bid combination does not include a bid from any bidder. A further round is needed.

## Bid evaluation

Scenario	Highest value (DKK million)	Highest-value feasible bid combinations	Exemption-compatible?	Possible winning bid combination?	
				Yes	No
$S_0$	225	No bids	Yes	Yes	No
$S_1$	225	No bids	No	-	-
		(H1)	No	-	-
		(H2)	No	-	-
		(H3)	No	-	-
$S_2$	225	No bids	No	-	-
		(H1)	No	-	-
		(H2)	No	-	-
		(H3)	No	-	-
		(H1, H2)	No	-	-

### Determining which lot categories require a price increment:

There are two lot categories. We run through the process described in the Information Memorandum as follows.

**Identifying omitted bidders:**

All three bidders are ‘omitted bidders’, as none of the bidders has its bid included in the single possible winning combination.

**Identifying categories that require a price increment by considering the headline bids from omitted bidders:**

We take each of the bidders in turn, in random order. Once we have run through this process for one bidder we may be able to skip some steps for the following bidder, if we already identified the need to increase the round price for some lot categories; specifically, in this example we would only need to go through this process for one bidder, as after considering one bidder we already identify the need to increase the round price for all the lot categories available. Below we describe the process for each of the bidders, as whether the process would be run for that bidder or not depends on the random order for considering bidders.

For any bidder, we go through each lot category included in that bidder’s headline bid in turn to create the hypothetical bids:

- We start with lot category B. We create a hypothetical bid where we retain the demand for B lots from the bidder’s headline bid and set the demand for C1 to zero. We then re-process bids using this bid instead of the bidder’s actual headline bid and check if the bidder continues to be an omitted bidder; if this is the case, then the round price for B lots must increase.
- We then consider lot category C1. We create a hypothetical bid where we retain the demand for C1 from the bidder’s headline bid and set the demand for B lots to zero. We then re-process bids using this bid instead of the bidder’s actual headline bid and check if the bidder continues to be an omitted bidder; if this is the case, then the round price for C1 lots must increase.

If the process above does not identify the need to increase round prices for any individual lot categories, we increase the round price for all the lot categories included in the bidder’s headline bid.

**Bidder 1:**

*B lots*

The hypothetical bid when only retaining B lots and setting the demand for C1 to zero is a bid of DKK 100 million for four B lots. Call this  $\hat{H}1^B$ .

We now have more feasible bid combinations:

- $S_0$ : In addition to the combination without any bids we can now accept a combination including only  $\hat{H}1^B$ ; this is also a highest-value bid combination, and is exemption-compatible as all feasible bid combinations are exemption-compatible in the scenario without exemptions.
- $S_1$ : In addition to the combinations that are feasible in  $S_0$  we can accept the combinations including each of H2 or H3 individually; and the combination of  $\hat{H}1^B$  along with H2 (however we cannot accept  $\hat{H}1^B$  along with H3, as we only have nine B lots). All of the feasible bid combinations are highest-value, as all bid combinations have the same value when the

round price is reserve; however, only one of these includes two non-zero bids and is thus exemption-compatible, namely the combination of  $\hat{H}1^B$  and H2.

- $S_2$ : We cannot accept any combinations that are not already feasible in  $S_1$ ; in particular we cannot accept any combinations including three bids because we do not have enough B lots. As in  $S_1$ , all of the feasible bid combinations are highest-value; however, none of these include three non-zero bids, which would be required for the combination to be exemption-compatible.

There are three possible winning bid combinations: not accepting any bids; accepting only  $\hat{H}1^B$ ; and accepting the pair ( $\hat{H}1^B$ , H2). Bidder 1 is still an omitted bidder as the bidder is not included in the combination that involves not accepting any bids. Therefore the price of B lots must increase.

Re-evaluation:

Scenario	Highest value (DKK million)	Highest-value feasible bid combinations	Exemption-compatible?	Possible winning bid combination?	Includes a bid from each bidder?
$S_0$	225	No bids	Yes	Yes	No
		( $\hat{H}1^B$ )	Yes	Yes	No
$S_1$	225	No bids	No	-	-
		( $\hat{H}1^B$ )	No	-	-
		(H2)	No	-	-
		(H3)	No	-	-
		( $\hat{H}1^B$ , H2)	Yes	Yes	No
$S_2$	225	No bids	No	-	-
		( $\hat{H}1^B$ )	No	-	-
		(H2)	No	-	-
		(H3)	No	-	-
		( $\hat{H}1^B$ , H2)	No	-	-

### C1 lots

The hypothetical bid when only retaining C1 and setting the demand for B lots to zero is a bid of DKK 0 for C1. Call this  $\hat{H}1^{C1}$ .

The feasible bid combinations are now as follows:

- $S_0$ : The only feasible combination is the combination without any bids. It is not possible to accept  $\hat{H}1^{C1}$  as we do not have any C1 lots.
- $S_1$ : In addition to the combination including no bids, we can accept the combinations including each of  $\hat{H}1^{C1}$ , H2 or H3 individually. We still cannot accept pairs of bids as we only have one C1 lot. None of the feasible bid combinations is exemption-compatible, as they include at most a single non-zero bid.
- $S_2$ : In addition to the combinations that are feasible in  $S_1$  we can also accept two pairs of bids:  $\hat{H}1^{C1}$  along with H2; and  $\hat{H}1^{C1}$  along with H3. We cannot accept H2 and H3 at the same time, or accept all three bids because we do not have enough B lots. None of the feasible bid

combinations includes three non-zero bids, which would be required for the combination to be exemption-compatible.

There is one single possible winning bid combination: not accepting any bids. Bidder 1 is still an omitted bidder as it is not included in the single possible winning bid combination. Therefore the price of C1 lots must increase.

Re-evaluation:

Scenario	Highest value (DKK million)	Highest-value feasible bid combinations	Exemption-compatible?	Possible winning bid combination?	Includes a bid from each bidder?
$S_0$	225	No bids	Yes	Yes	No
$S_1$	225	No bids	No	-	-
		$(\hat{H}1^{C1})$	No	-	-
		$(H2)$	No	-	-
$S_2$	225	$(H3)$	No	-	-
		No bids	No	-	-
		$(\hat{H}1^{C1})$	No	-	-
		$(H2)$	No	-	-
		$(H3)$	No	-	-
		$(\hat{H}1^{C2}, H2)$	No	-	-
$(\hat{H}1^{C2}, H3)$	No	-	-		

*Need to increase the price of all lots included in the bid?*

As the process above has already identified the need to increase the price for all of the available lot categories we do not need to go through the process of checking the need for increasing the price of all lot categories included in the bidder's headline bid.

**Bidder 2:**

The assessment is analogous as for Bidder 1 as these two bidders have submitted identical bids.

**Bidder 3:**

*B lots*

The hypothetical bid when only retaining B lots and setting the demand for C1 to zero is a bid of DKK 150 million for six B lots. Call this  $\hat{H}3^B$ .

As with Bidder 1, we now have more feasible bid combinations:

- $S_0$ : In addition to the combination without any bids it is now also possible to accept the combination consisting of  $\hat{H}3^B$  alone; this is also a highest-value bid combination, and is exemption-compatible as all feasible bid combinations are exemption-compatible in  $S_0$ .
- $S_7$ : In addition to the combinations that are feasible in  $S_0$  we can also accept the combinations including each of H1, H2 or  $\hat{H}3^B$  individually. However, unlike when considering Bidder 1, we cannot accept  $\hat{H}3^B$  along with H1 or H2 (as we only have nine B lots); it is still not possible to accept H1 along with H2 as we do not have enough C1

lots. None of the feasible bid combinations is exemption-compatible, as they at most include one non-zero bid.

- $S_2$ : In addition to the combinations that are feasible in  $S_1$  we can accept H1 along with H2; however, we cannot accept  $\hat{H}3^B$  along any other bids (and we cannot accept the three bids), as we do not have enough B lots. As with the original set of bids, none of the highest-value feasible bid combinations is exemption-compatible, as we none of them includes three non-zero bids.

There are two possible winning bid combinations: not accepting any bids; and accepting only  $\hat{H}3^B$ . Bidder 3 is still an omitted bidder as it is not included in the combination that involves not accepting any bids. Therefore the price of B lots must increase.

Re-evaluation:

Scenario	Highest value (DKK million)	Highest-value feasible bid combinations	Exemption-compatible?	Possible winning bid combination?	Includes a bid from each bidder?
$S_0$	225	No bids	Yes	Yes	No
		$(\hat{H}3^B)$	Yes	Yes	No
$S_1$	225	No bids	No	-	-
		(H1)	No	-	-
		(H2)	No	-	-
		$(\hat{H}3^B)$	No	-	-
$S_2$	225	No bids	No	-	-
		$(\hat{H}1^B)$	No	-	-
		(H2)	No	-	-
		(H3)	No	-	-
		(H1, H2)	No	-	-

### C1 lots

The hypothetical bid when only retaining C1 and setting the demand for B lots to zero is a bid of DKK 0 for C1. Call this  $\hat{H}3^{C1}$ .

The feasible bid combinations are now as follows:

- $S_0$ : It is only possible to accept the combination without any bids. It is not possible to accept  $\hat{H}3^{C1}$  as we do not have any C1 lots.
- $S_1$ : In addition to the combination with no bids, we can also accept the combinations including each of the bids H1, H2 or  $\hat{H}3^{C1}$  individually. We cannot accept pairs of bids as we only have one C1 lot. None of the feasible bid combinations is exemption-compatible, as they include at most one single non-zero bid.
- $S_2$ : In addition to the combinations that are feasible in  $S_1$  we can also accept all pairs of bids: H1 along H2; H1 along  $\hat{H}3^{C1}$ ; and H2 along  $\hat{H}3^{C1}$ . We are still unable to accept a combination of all three bids, as we only have two C1 lots. None of the feasible bid combinations are exemption-compatible as they at most include two non-zero bids.

There is a single possible winning bid combination: not accepting any bids. Bidder 3 is still an omitted bidder as it is not included in the single possible winning bid combination. Therefore the price of C1 lots must increase.

Re-evaluation:

Scenario	Highest value (DKK million)	Highest-value feasible bid combinations	Exemption-compatible?	Possible winning bid combination?	Includes a bid from each bidder?
S <sub>0</sub>	225	No bids	Yes	Yes	No
S <sub>1</sub>	225	No bids	No	-	-
		(H1)	No	-	-
		(H2)	No	-	-
		(H3 <sup>C1</sup> )	No	-	-
S <sub>2</sub>	225	No bids	No	-	-
		(H1)	No	-	-
		(H2)	No	-	-
		(H3 <sup>C1</sup> )	No	-	-
		(H1, H2)	No	-	-
		(H1, H3 <sup>C1</sup> )	No	-	-
		(H2, H3 <sup>C1</sup> )	No	-	-

*Need to increase the price of all lots included in the bid?*

As the process above has identified the need to increase the price for all of the available lot categories we do not need to go through the process of checking the need for increasing the price of all lot categories for which the bidder has bid.

### Summary

Omitted bidders	Lot categories identified as requiring a price increment (i.e. bidder still omitted when using hypothetical bid only including lots in this category)		
	B	C1	Resort to increasing price for all lots in package
Bidder 1	Yes	Yes	No
Bidder 2	Yes	Yes	No
Bidder 3	Yes	Yes	No

For the next round we need to increase the price for both B lots and C1 lots.

## Example 11

Building on Example 10, suppose that in a subsequent round the price of B lots is DKK 26 million and the price for C1 lots is DKK 10 million. Suppose that all three bidders have continued to make headline bids for the same package and have made no additional bids.

As round prices are now above reserve, assigning lots through accepting bids yields a higher value than leaving them unsold. This does not affect the assessment of whether the auction can close, as there are no exemption-compatible bid combinations in  $S_1$  and  $S_2$  and thus we continue to have a single possible winning bid combination from  $S_0$ , which involves not accepting any bids. However, this affects the assessment of which lot categories need a price increase.

Specifically, whilst all three bidders are omitted when using the original bids, they are no longer omitted when we consider a hypothetical bid that only includes B lots for each of the bidders in turn, whilst at the same time they continue to be omitted when we consider a hypothetical bid that only includes C1; therefore, for each bidder we identify the need to increase the price for C1 lots but not for B lots. As a result, we only increase the price of C1 for the following round.

Bids (round price for B lots is DKK 26 million, and for C1 lots DKK 10 million)

Bid id	Bidder	Headline/additional	B lots	C1	Amount (DKK million)
H1	Bidder1	Headline	4	1	114
H2	Bidder2	Headline	4	1	114
H3	Bidder3	Headline	6	1	166

## Bid evaluation

Scenario	Highest value (DKK million)	Highest-value feasible bid combinations	Exemption-compatible?	Possible winning bid combination?	Includes a bid from each bidder?
$S_0$	225	No bids	Yes	Yes	No
$S_1$	241	(H3)	No	-	-
$S_2$	253	(H1, H2)	No	-	-

## Price increments

Omitted bidders: Bidder 1, Bidder 2 and Bidder 3.

**Bidder 1***B lots*

Bids:

Bid id	Bidder	Headline/additional/ hypothetical	B lots	C1	Amount (DKK million)
H1 <sup>B</sup>	Bidder1	Hypothetical	4	0	104
H2	Bidder2	Headline	4	1	114
H3	Bidder3	Headline	6	1	166

Re-evaluation:

Scenario	Highest value (DKK million)	Highest- value feasible bid combinations	Exemption- compatible?	Possible winning bid combination?	Includes a bid from the bidder?
S <sub>0</sub>	229	(H1 <sup>B</sup> )	Yes	No	-
S <sub>1</sub>	243	(H1 <sup>B</sup> , H2)	Yes	Yes	Yes
S <sub>2</sub>	243	(H1 <sup>B</sup> , H2)	No	-	-

Bidder is not omitted upon re-evaluation, as its hypothetical bid is included in the single possible winning combination. Therefore, on the basis of this bid we do not identify a need to increase the price of B lots.

*C1 lots*

Bids:

Bid id	Bidder	Headline/additional/ hypothetical	B lots	C1	Amount (DKK million)
H1 <sup>C1</sup>	Bidder1	Hypothetical	0	1	10
H2	Bidder2	Headline	4	1	114
H3	Bidder3	Headline	6	1	166

Re-evaluation:

Scenario	Highest value (DKK million)	Highest- value feasible bid combinations	Exemption- compatible?	Possible winning bid combination?	Includes a bid from the bidder?
S <sub>0</sub>	225	No bids	Yes	Yes	No
S <sub>1</sub>	241	(H3)	No	-	-
S <sub>2</sub>	251	(H1 <sup>C1</sup> , H3)	No	-	-

Bidder is still omitted upon re-evaluation. The round price of C1 lots must increase for the following round.

*Need to increase the price of all lots included in the bid?*

No, as we identified the need to increase the price of C1 lots.

**Bidder 2:**

The assessment is analogous as for Bidder 1 as these two bidders have submitted identical bids.



**Bidder 3***B lots*

Bids:

Bid id	Bidder	Headline/additional/ hypothetical	B lots	C1	Amount (DKK million)
H1	Bidder1	Headline	4	1	114
H2	Bidder2	Headline	4	1	114
H3 <sup>B</sup>	Bidder3	Hypothetical	6	0	156

Re-evaluation:

Scenario	Highest value (DKK million)	Highest- value feasible bid combinations	Exemption- compatible?	Possible winning bid combination?	Includes a bid from the bidder?
S <sub>0</sub>	231	(H3 <sup>B</sup> )	Yes	Yes	Yes
S <sub>1</sub>	239	(H1)	No	-	-
		(H2)	No	-	-
S <sub>2</sub>	253	(H1, H2)	No	-	-

Bidder is not omitted upon re-evaluation, as its hypothetical bid is included in the single possible winning combination. Therefore, on the basis of this bid we do not identify a need to increase the price of B lots.

*C1 lots*

Bids:

Bid id	Bidder	Headline/additional/ hypothetical	B lots	C1	Amount (DKK million)
H1	Bidder1	Headline	4	1	114
H2	Bidder2	Headline	4	1	114
H3 <sup>C1</sup>	Bidder3	Hypothetical	0	1	10

Re-evaluation:

Scenario	Highest value (DKK million)	Highest- value feasible bid combinations	Exemption- compatible?	Possible winning bid combination?	Includes a bid from the bidder?
S <sub>0</sub>	225	No bids	Yes	Yes	No
S <sub>1</sub>	239	(H1)	No	-	-
		(H2)	No	-	-
S <sub>2</sub>	253	(H1, H2)	No	-	-

Bidder is still omitted upon re-evaluation. The round price of C1 lots must increase for the following round.

*Need to increase the price of all lots included in the bid?*

No, as we identified the need to increase the price of C1 lots.

**Summary**

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Lot categories identified as requiring a price increment (i.e. bidder still omitted when using hypothetical bid only including lots in this category)

Omitted bidders	B	C1	Resort to increasing price for all lots in package
Bidder 1	No	Yes	No
Bidder 2	No	Yes	No
Bidder 3	No	Yes	No

---

For the next round we need to increase the price for C1 lots only.

## Example 12

In this example a further round is needed, but only the price of B lots must increase.

Again building on example 10, suppose that the price of B lots has increased to DKK 26 million and the price for C1 lots to DKK 10 million.

Suppose that after the first round, Bidder 1 and Bidder 2 contract their demand for B lots whilst they maintain their demand for C1, both switching to a package of three B lots and C1. By contrast, Bidder 3 maintains its demand for B lots but gives up bidding on C1, so it switches to the package of six B lots. Bidders do not bid for any other packages, and do not increase their bids from the first round, so the only bids for consideration are their headline bids in the current round and their headline bids in the first round (which are now entering the evaluation as additional bids in the table below).

In this situation, a further round is still needed, as it is not possible to accept a bid from each bidder. However, when checking for which lot categories prices need to increase:

- Bidder 3 is not an omitted bidder; and
- when assessing the need to increase prices on the basis of the headline bids from Bidder 1 and Bidder 2, we only identify C1 as requiring a price increment.

Bids (round price for B lots is DKK 26 million, and for C1 lots DKK 10 million)

Bid id	Bidder	Headline/additional	B lots	C1	Amount (DKK million)
H1	Bidder1	Headline	3	1	88
A1	Bidder1	Additional	4	1	100
H2	Bidder2	Headline	3	1	88
A2	Bidder2	Additional	4	1	100
H3	Bidder3	Headline	6	0	156
A3	Bidder3	Additional	6	1	150

### Bid evaluation

Scenario	Highest value (DKK million)	Highest-value feasible bid combinations	Exemption-compatible?	Possible winning bid combination?	Includes a bid from each bidder?
S <sub>0</sub>	231	(H3)	Yes	No	-
S <sub>1</sub>	244	(H1, H3)	Yes	Yes	No
		(H2, H3)	Yes	Yes	No
S <sub>2</sub>	251	(H1, H2)	No	-	-

### Price increments

Omitted bidders: Bidder 1 and Bidder 2.

Omitted bidders	Lot categories identified as requiring a price increment (i.e. bidder still omitted when using hypothetical bid only including lots in this category)			Resort to increasing price for all lots in package
	B	C1		
Bidder 1	Yes	No		No
Bidder 2	Yes	No		No

For the next round we need to increase the price for B lots only.

### Example 13

In this example there are exemption-compatible combinations in  $S_7$ , but these do not achieve the highest value for this scenario; because no highest-value feasible combinations in  $S_7$  are exemption-compatible, the list of possible winning bid combinations only includes combinations from  $S_0$ .

As in the preceding example, suppose that the price of B lots has increased to DKK 26 million and the price for C1 lots has increased to DKK 10 million. However, we assume that Bidder 1 has been bidding for the package of three B lots and C1 since the beginning of the auction (and has made no additional bids); Bidder 2 has been bidding for the package of four B lots and C1 since the beginning of the auction (and has made no additional bids); and Bidder 3 has been bidding on the package of six B lots and C1, but has also made an additional bid for the package of three B lots at reserve.

A further round is needed. When checking for which lot categories prices need to increase, we find that the price for all lot categories needs to increase, as Bidder 3's bid does not conflict in relation to a specific lot category, but only in relation to the combination of lots demanded: either its demand for B lots or its demand for C1 lots could be accommodated, but not both at the same time. Thus the price for all the lots included in bidder 3's headline bid needs to increase for the following round.

Bids (round price for B lots is DKK 26 million, and for C1 lots DKK 10 million)

Bid id	Bidder	Headline/additional	B lots	C1	Amount (DKK million)
H1	Bidder1	Headline	3	1	88
H2	Bidder2	Headline	4	1	114
H3	Bidder3	Headline	6	1	166
A3	Bidder3	Additional	3	0	75

### Bid evaluation

Scenario	Highest value (DKK million)	Highest-value feasible bid combinations	Exemption-compatible?	Possible winning bid combination?	Includes a bid from each bidder?
$S_0$	225	No bids (A3)	Yes	Yes	No
$S_1$	241	(H3)	No	-	-
$S_2$	254	(H1, H3)	No	-	-

### Price increments

Omitted bidders: Bidder 1, Bidder 2 and Bidder 3.

When checking the need to increase prices by looking at the headline bids submitted by Bidder 1 and Bidder 2 we identify the need to increase the price of C1 when considering that lot category individually (i.e. when re-evaluating bids using a hypothetical bid that only retains the demand for C1 lots in the headline bid).

However, when considering Bidder 3 we do not identify the need to increase the price of any lot category when considering them individually. This is because Bidder 3's headline bid does not conflict with the demand of other bidders on either of the categories individually, but only as a whole. Given that this situation is slightly different than in the previous examples, we provide details on how we check for the need to increase prices when considering the headline bid from Bidder 3.

### Bidder 3

#### B lots

Bids:

Bid id	Bidder	Headline/additional/		C1	Amount (DKK million)
		hypothetical	B lots		
H1	Bidder1	Headline	3	1	88
H2	Bidder2	Headline	4	1	114
H3 <sup>B</sup>	Bidder3	Hypothetical	6	0	156
A3	Bidder3	Additional	3	0	75

Re-evaluation:

Scenario	Highest value (DKK million)	Highest-value feasible bid combinations	Exemption-compatible?	Possible winning bid combination?	Includes a bid from each bidder?
S <sub>0</sub>	231	(H3 <sup>B</sup> )	Yes	No	-
S <sub>1</sub>	244	(H1, H3 <sup>B</sup> )	Yes	Yes	No
S <sub>2</sub>	252	(H1, H2)	No	-	-

Bidder 3 is not omitted upon re-evaluation, and thus we do not identify a need to increase the price for B lots on the basis of this check.

*C1 lots*

Bids:

Bid id	Bidder	Headline/additional/ hypothetical	B lots	C1	Amount (DKK million)
H1	Bidder1	Headline	3	1	88
H2	Bidder2	Headline	4	1	114
H3 <sup>C1</sup>	Bidder3	Hypothetical	0	1	10
A3	Bidder3	Additional	3	0	75

Re-evaluation:

Scenario	Highest value (DKK million)	Highest-value feasible bid combinations	Exemption-compatible?	Possible winning bid combination?	Includes a bid from each bidder?
S <sub>0</sub>	225	No bids	Yes	No	-
		(A3)	Yes	No	-
S <sub>1</sub>	239	(H2)	Yes	No	-
		(H2, A3)	Yes	Yes	No
S <sub>2</sub>	252	(H1, H2)	No	-	-

Bidder 3 is not omitted upon re-evaluation, and thus we do not identify a need to increase the price for C1 lots on the basis of this check.

*Need to increase the price of all lots included in the bid?*

As none of the lot categories are identified as needing a price increase when considered individually, we need to increase the price of all lot categories included in Bidder 3's headline bid.

**Summary**

Omitted bidders	Lot categories identified as requiring a price increment (i.e. bidder still omitted when using hypothetical bid only including lots in this category)			Resort to increasing price for all lots in package
	B	C1		
Bidder 1	No	Yes		No
Bidder 2	No	Yes		No
Bidder 3	No	No		Yes

For the next round we need to increase the price for all lot categories.

## Example 14

This example is similar to the previous one, but whilst in the previous example the exemption-compatible bid combination in  $S_1$  was outbid by Bidder 3's headline bid, now the exemption-compatible bid combination in  $S_1$  is outbid by a combination of bids that includes the headline bid from either Bidder 1 or Bidder 2. As before, this results in the list of possible winning bid combinations only including combinations from  $S_0$ .

As in the preceding example, suppose that the price of B lots has increased to DKK 26 million and the price for C1 lots has increased to DKK 10 million. However, we assume that both Bidder 1 and Bidder 2 have been bidding for the package of four B lots and C1 since the beginning of the auction (and have made no additional bids); and Bidder 3 has been bidding on the package of two B lots and C1, and has also made an additional bid for the package of six B lots at reserve.

A further round is needed. When checking which lot categories need a price increase, we only identify the need to increase the price for C1.

Bids (round price for B lots is DKK 26 million, and for C1 lots DKK 10 million)

Bid id	Bidder	Headline/additional	B lots	C1	Amount (DKK million)
H1	Bidder1	Headline	4	1	114
H2	Bidder2	Headline	4	1	114
H3	Bidder3	Headline	2	1	62
A3	Bidder3	Additional	6	0	150

## Bid evaluation

Scenario	Highest value (DKK million)	Highest-value feasible bid combinations	Exemption-compatible?	Possible winning bid combination?	Includes a bid from each bidder?
$S_0$	225	No bids	Yes	Yes	No
		(A3)	Yes	Yes	No
$S_1$	239	(H1)	No	-	-
		(H2)	No	-	-
$S_2$	253	(H1, H2)	No	-	-

## Price increments

Omitted bidders: Bidder 1, Bidder 2 and Bidder 3.



Examples of bid evaluation in the third auction stage

Lot categories identified as requiring a price increment (i.e. bidder still omitted when using hypothetical bid only including lots in this category)			
Omitted bidders	B	C1	Resort to increasing price for all lots in package
Bidder 1	No	Yes	No
Bidder 2	No	Yes	No
Bidder 3	No	Yes	No

For the next round we need to increase the price for C1 lots only.

## Example 15

This example shows a situation where there are highest-value feasible bid combinations in  $S_1$  that are exemption-compatible and are possible winning combinations; however, these do not include a bid from each bidder. Therefore, a further round is needed.

When checking which lot categories require a price increase, only Bidder 1 and Bidder 2 are omitted. We identify the need to increase the price for C1 only.

Bids (round price for B lots is DKK 26 million, and for C1 lots DKK 10 million)

Bid id	Bidder	Headline/additional	B lots	C1	Amount (DKK million)
H1	Bidder1	Headline	4	1	114
A1	Bidder1	Additional	3	1	80
H2	Bidder2	Headline	4	1	114
A2	Bidder2	Additional	3	1	85
H3	Bidder3	Headline	2	1	62
A3	Bidder3	Additional	4	0	100

## Bid evaluation

Scenario	Highest value (DKK million)	Highest-value feasible bid combinations	Exemption-compatible?	Possible winning bid combination?	Includes a bid from each bidder?
S <sub>0</sub>	225	No bids	Yes	No	-
		(A3)	Yes	No	-
		(H1)	No	-	-
S <sub>1</sub>	239	(H2)	No	-	-
		(H1, A3)	Yes	Yes	No
		(H2, A3)	Yes	Yes	No
S <sub>2</sub>	253	(H1, H2)	No	-	-

## Price increments

Omitted bidders: Bidder 1 and Bidder 2.

Omitted bidders	Lot categories identified as requiring a price increment (i.e. bidder still omitted when using hypothetical bid only including lots in this category)		
	B	C1	Resort to increasing price for all lots in package
Bidder 1	No	Yes	No
Bidder 2	No	Yes	No

For the next round we need to increase the price for C1 lots only.

## Example 16

In this example Bidder 1 and Bidder 2 submit a zero headline bid, and the third auction stage can end with a winning outcome in  $S_0$ .

Bids (round price for B lots is DKK 26 million, and for C1 lots DKK 10 million)

Bid id	Bidder	Headline/additional	B lots	C1	Amount (DKK million)
H1	Bidder1	Headline	0	0	0
A1	Bidder1	Additional	4	1	109
H2	Bidder2	Headline	0	0	0
A2	Bidder2	Additional	4	1	109
H3	Bidder3	Headline	6	1	166
A3	Bidder3	Additional	6	0	150

## Bid evaluation

Scenario	Highest value (DKK million)	Highest-value feasible bid combinations	Exemption-compatible?	Possible winning bid combination?	Includes a bid from each bidder?
$S_0$	225	No bids	Yes	Yes	No
		(H1)	Yes	Yes	No
		(H2)	Yes	Yes	No
		(A3)	Yes	Yes	No
		(H1, H2)	Yes	Yes	No
		(H1, A3)	Yes	Yes	No
		(H2, A3)	Yes	Yes	No
		(H1, H2, A3)	Yes	Yes	Yes
$S_1$	241	(H3)	No	-	-
		(H1, H3)	No	-	-
		(H2, H3)	No	-	-
		(H1, H2, H3)	No	-	-
$S_2$	243	(H1, H2)	No	-	-

## Example 17

This example shows a situation in which Bidder 1 submits a zero headline bid, and the third auction stage can end with a winning outcome in  $S_1$ .

Bids (round price for B lots is DKK 26 million, and for C1 lots DKK 10 million)

Bid id	Bidder	Headline/additional	B lots	C1	Amount (DKK million)
H1	Bidder1	Headline	0	0	0
A1	Bidder1	Additional	4	1	105
H2	Bidder2	Headline	2	1	62
A2	Bidder2	Additional	3	1	80
H3	Bidder3	Headline	2	1	62
A3	Bidder3	Additional	4	0	100

## Bid evaluation

Scenario	Highest value (DKK million)	Highest-value feasible bid combinations	Exemption-compatible?	Possible winning bid combination?	Includes a bid from each bidder?
$S_0$	225	No bids	Yes	No	-
		(H1)	Yes	No	-
		(A3)	Yes	No	-
		(H1, A3)	Yes	No	-
		(H2)	No	-	-
$S_1$	237	(H3)	No	-	-
		(H1, H2)	No	-	-
		(H1, H3)	No	-	-
		(H2, A3)	Yes	Yes	No
		(H1, H2, A3)	Yes	Yes	Yes
$S_2$	249	(H2, H3)	No	-	-
		(H1, H2, H3)	No	-	-

## Example 18

Finally, this example shows a situation in which none of the bidders has submitted a zero headline bid, and the third auction stage can end with a winning outcome in  $S_2$ .

Bids (round price for B lots is DKK 26 million, and for C1 lots DKK 10 million)

Bid id	Bidder	Headline/additional	B lots	C1	Amount (DKK million)
H1	Bidder1	Headline	3	1	88
A1	Bidder1	Additional	4	1	105
H2	Bidder2	Headline	2	1	62
A2	Bidder2	Additional	3	1	80
H3	Bidder3	Headline	2	1	62
A3	Bidder3	Additional	4	0	100

### Bid evaluation

Scenario	Highest value (DKK million)	Highest-value feasible bid combinations	Exemption-compatible?	Possible winning bid combination?	Includes a bid from each bidder?
$S_0$	225	No bids	Yes	No	-
		(A3)	Yes	No	-
$S_1$	238	(H1)	No	-	-
		(H1, A3)	Yes	No	-
		(H1, H2)	No	-	-
$S_2$	250	(H1, H3)	No	-	-
		(H1, H2, A3)	Yes	Yes	Yes

## 4 Examples with all three exemption lot categories

We now present some examples for the case in which there are three exemption lot categories. This will happen if none of the A lots are assigned in the first auction stage. In this case, 13 B lots are offered in the third auction stage, and all three exemption lot categories (one for each of the three coverage area groups, lot categories C1, C2 and C3) are available.

The process for assessing bids is the same as in the case with only one exemption lot category, except that when checking for feasible bid combinations we need to take four lot categories into account instead of two; for this reason, we skip the detailed explanation of this process and directly provide the results table as in the above examples.

The process for identifying the lot categories that require a price increment when considering the bid from an omitted bidder may now involve an additional step. In particular, if we do not identify the need to increase the price for any of the lot categories included in the bidder's headline bids when considered individually

(i.e. if the bidder is no longer omitted when considering hypothetical bids that retain the demand for one lot category only), we then investigate whether the headline bid clashes due to the demand in exemption lots across all exemption categories. We do this by considering a hypothetical bid that retains the demand in the headline bid in all exemption lot categories but not for B lots (no B lots are included in this hypothetical bid). If the bidder is still omitted when considering this hypothetical bid, then we need to increase the price of all exemption lot categories included in the bidder's headline bid. However, if the bidder is not omitted when considering this hypothetical, we must resort to increasing the price for all the lot categories included in the bid. When this check is not required, this is indicated with 'NA' (Not Applicable). This step is rarely required, as in most cases we identify the need for price increases when looking at individual lot categories; however, there are some cases where this additional check is used, as shown in example 26 below.

## Example 19

We start with a very simple example, in which three bidders make identical headline bids in the first round, for six B lots and an exemption in each of the exemption lot categories, and no additional bids.

Clearly, it is not possible to accommodate all three bids. The only possible winning bid combination does not include any bids, and thus a further round is needed. When checking which lot categories need a price increment for the following round, we identify the need to increase the price for all categories individually.

Bids (round price for B lots is DKK 25 million, and for C1, C2 and C3 lots is DKK 0)

Bid id	Bidder	Headline/additional	B	C1	C2	C3	Amount (DKK million)
H1	Bidder1	Headline	6	1	1	1	150
H2	Bidder2	Headline	6	1	1	1	150
H3	Bidder3	Headline	6	1	1	1	150

## Bid evaluation

Scenario	Highest value (DKK million)	Highest-value feasible bid combinations	Exemption- compatible?	Possible winning bid combination?	Includes a bid from each bidder?
S <sub>0</sub>	325	No bids	Yes	Yes	No
S <sub>1</sub>	325	No bids	No	-	-
		(H1)	No	-	-
		(H2)	No	-	-
		(H3)	No	-	-
S <sub>2</sub>	325	No bids	No	-	-
		(H1)	No	-	-
		(H2)	No	-	-
		(H3)	No	-	-
		(H1, H2)	No	-	-
		(H1, H3)	No	-	-
		(H2, H3)	No	-	-

## Price increments

Omitted bidders: Bidder 1, Bidder 2 and Bidder 3.

Examples of bid evaluation in the third auction stage

Omitted bidders	Lot categories identified as requiring a price increment (i.e. bidder still omitted when using hypothetical bid only including lots in this category)				All exemption lots included	
	B	C1	C2	C3	lots included	All lots included
Bidder 1	Yes	Yes	Yes	Yes	NA	NA
Bidder 2	Yes	Yes	Yes	Yes	NA	NA
Bidder 3	Yes	Yes	Yes	Yes	NA	NA

For the next round we need to increase the price for all lots.



## Example 20

Building on the previous example, suppose that the price of B lots increases to DKK 26 million and the price for exemption lots to DKK 1 million in all exemption lot categories. Suppose that all three bidders continue to make headline bids for the same package and do not submit any additional bids.

As before, it is not possible to accommodate all three bids and the only possible winning bid combination is not to include any bids; thus a further round is needed. However, the number of highest-value feasible bid combinations in scenarios with exemptions is smaller, as assigning lots results in higher value than leaving them unassigned and valued at reserve. As a consequence, when checking which lot categories need a price increase, we identify the need to increase the price for all exemption lot categories individually, but not for B lots.

Bids (round price for B lots is DKK 26 million, and for C1, C2 and C3 lots is DKK 1 million)

Bid id	Bidder	Headline/additional	B	C1	C2	C3	Amount (DKK million)
H1	Bidder1	Headline	6	1	1	1	159
H2	Bidder2	Headline	6	1	1	1	159
H3	Bidder3	Headline	6	1	1	1	159

## Bid evaluation

Scenario	Highest value (DKK million)	Highest-value feasible bid combinations	Exemption-compatible?	Possible winning bid combination?	Includes a bid from each bidder?
S <sub>0</sub>	325	No bids	Yes	Yes	No
S <sub>1</sub>	334	(H1)	No	-	-
		(H2)	No	-	-
		(H3)	No	-	-
S <sub>2</sub>	343	(H1, H2)	No	-	-
		(H1, H3)	No	-	-
		(H2, H3)	No	-	-

## Price increments

Omitted bidders: Bidder 1, Bidder 2 and Bidder 3.

Examples of bid evaluation in the third auction stage

Omitted bidders	Lot categories identified as requiring a price increment (i.e. bidder still omitted when using hypothetical bid only including lots in this category)				All exemption lots included	
	B	C1	C2	C3		All lots included
Bidder 1	No	Yes	Yes	Yes	NA	NA
Bidder 2	No	Yes	Yes	Yes	NA	NA
Bidder 3	No	Yes	Yes	Yes	NA	NA

For the next round we need to increase the price for all exemption lot categories but not for B lots.

## Example 21

Building on the previous example, suppose that the price of B lots has remained at DKK 26 million, whilst the price for exemption lots has increased to DKK 10 million in all exemption lot categories. Suppose that all three bidders have only made headline bids for the same package, and no additional bids, up to this point. At these prices, suppose that bidders keep making a headline bid for the same package, but also submit additional bids for each permissible package that includes six B lots and two exemption lots (effectively offering to take on the coverage obligation in any of the area groups, but only in one group) at reserve.

It is still not possible to accommodate a bid from each bidder. Whilst there are possible combinations of bids that do not conflict in relation to exemptions (those involving assigning each of the coverage area group to a different bidder), the bids in the combination would still conflict in relation to B lots. In addition, as the additional bids are at reserve, they do not provide a better alternative than accepting any of the headline bids in the scenarios in which exemption lots are available, and thus the combinations that involve additional bids are not shortlisted as possible winning combinations.

When checking which lot categories require a price increase we still identify the need to increase the price for all exemption lot categories individually, but not for B lots.

Bids (round price for B lots is DKK 26 million, and for C1, C2 and C3 lots is DKK 10 million)

Bid id	Bidder	Headline/additional	B	C1	C2	C3	Amount (DKK million)
H1	Bidder1	Headline	6	1	1	1	186
A1.1	Bidder1	Additional	6	1	1	0	150
A1.2	Bidder1	Additional	6	1	0	1	150
A1.3	Bidder1	Additional	6	0	1	1	150
H2	Bidder2	Headline	6	1	1	1	186
A2.1	Bidder2	Additional	6	1	1	0	150
A2.2	Bidder2	Additional	6	1	0	1	150
A2.3	Bidder2	Additional	6	0	1	1	150
H3	Bidder3	Headline	6	1	1	1	186
A3.1	Bidder3	Additional	6	1	1	0	150
A3.2	Bidder3	Additional	6	1	0	1	150
A3.3	Bidder3	Additional	6	0	1	1	150

### Bid evaluation

Scenario	Highest value (DKK million)	Highest-value feasible bid combinations	Exemption-compatible?	Possible winning bid combination?	Includes a bid from each bidder?
S <sub>0</sub>	325	No bids	Yes	Yes	No
S <sub>1</sub>	361	(H1)	No	-	-
		(H2)	No	-	-
		(H3)	No	-	-
S <sub>2</sub>	397	(H1, H2)	No	-	-
		(H1, H3)	No	-	-
		(H2, H3)	No	-	-

### Price increments

Omitted bidders: Bidder 1, Bidder 2 and Bidder 3.

Omitted bidders	Lot categories identified as requiring a price increment (i.e. bidder still omitted when using hypothetical bid only including lots in this category)				All exemption lots included	All lots included
	B	C1	C2	C3		
Bidder 1	No	Yes	Yes	Yes	NA	NA
Bidder 2	No	Yes	Yes	Yes	NA	NA
Bidder 3	No	Yes	Yes	Yes	NA	NA

For the next round we need to increase the price for all exemption lot categories but not for B lots.

## Example 22

This is a variation of the previous example in which bidders do not submit identical additional bids but rather show different willingness to accept the coverage obligation. As before we assume that the price for exemption lots has increased to DKK 10 million in all exemption lot categories, and that up to this point all three bidders have only made headline bids for the package of six B lots and exemption lots in all categories, and no additional bids.

At current round prices, all bidders maintain the package of their headline bid, but also submit additional bids at reserve. Bidder 1 makes an additional bid for a package with five B lots and no exemptions; Bidder 2 makes additional bids for a package with five B lots and exemptions in all categories, and for each permissible package that includes five B lots and two exemption lots (effectively offering to take on the coverage obligation in any of the area groups, but only in one group); and Bidder 3 makes an additional bid for a package with three B lots and exemption lots in all categories.

It would now be possible to accommodate one additional bid from each bidder. However, as additional bids are at reserve, any such combination fails to achieve the highest value in  $S_2$ , where accepting two headline bids yields a higher value. Therefore, none of the combinations that include a bid from each bidder is shortlisted as a possible winning bid combination.

When checking for which lot categories prices for the following round need to increase, only Bidder 2 and Bidder 3 are omitted. Again we only identify the need to increase the price for all exemption lot categories individually, but not for B lots.

Bids (round price for B lots is DKK 26 million, and for C1, C2 and C3 lots is DKK 10 million)

Bid id	Bidder	Headline/additional	B	C1	C2	C3	Amount (DKK million)
H1	Bidder1	Headline	6	1	1	1	186
A1	Bidder1	Additional	5	0	0	0	125
H2	Bidder2	Headline	6	1	1	1	186
A2.1	Bidder2	Additional	5	1	1	1	125
A2.2	Bidder2	Additional	5	1	1	0	125
A2.3	Bidder2	Additional	5	1	0	1	125
A2.4	Bidder2	Additional	5	0	1	1	125
H3	Bidder3	Headline	6	1	1	1	186
A3	Bidder3	Additional	3	1	1	1	75

### Bid evaluation

Scenario	Highest value (DKK million)	Highest-value feasible bid combinations	Exemption-compatible?	Possible winning bid combination?	Includes a bid from each bidder?
S <sub>0</sub>	325	No bids	Yes	No	-
		(A1)	Yes	No	-
		(H1)	No	-	-
		(H2)	No	-	-
S <sub>1</sub>	361	(H3)	No	-	-
		(A1, H2)	Yes	Yes	No
		(A1, H3)	Yes	Yes	No
		(H1, H2)	No	-	-
S <sub>2</sub>	397	(H1, H3)	No	-	-
		(H2, H3)	No	-	-

### Price increments

Omitted bidders: Bidder 2 and Bidder 3.

Omitted bidders	Lot categories identified as requiring a price increment (i.e. bidder still omitted when using hypothetical bid only including lots in this category)					All exemption lots included	All lots included
	B	C1	C2	C3	All exemption lots included		
Bidder 2	No	Yes	Yes	Yes	NA	NA	
Bidder 3	No	Yes	Yes	Yes	NA	NA	

For the next round we need to increase the price for all exemption lot categories but not for B lots.

## Example 23

Using the same framework as in the previous example, we show a situation in which the third auction stage would end. The only difference relative to the previous example is that bidders offer different bid amounts in the additional bids: additional bids are now made at round prices (hence the maximum allowed) rather than at reserve (the minimum allowed).

In this case accepting additional bids from all bidders yields the highest value, and the third auction stage can end.

Bids (round price for B lots is DKK 26 million, and for C1, C2 and C3 lots is DKK 10 million)

Bid id	Bidder	Headline/additiona l	B	C 1	C 2	C 3	Amount (DKK million)
H1	Bidder 1	Headline	6	1	1	1	186
A1	Bidder 1	Additional	5	0	0	0	130
H2	Bidder 2	Headline	6	1	1	1	186
A2.1	Bidder 2	Additional	5	1	1	1	160
A2.2	Bidder 2	Additional	5	1	1	0	150
A2.3	Bidder 2	Additional	5	1	0	1	150
A2.4	Bidder 2	Additional	5	0	1	1	150
H3	Bidder 3	Headline	6	1	1	1	186
A3	Bidder 3	Additional	3	1	1	1	108

## Bid evaluation

Scenario	Highest value (DKK million)	Highest- value feasible bid combinations	Exemption- compatible?	Possible winning bid combination?	Includes a bid from each bidder?
S <sub>0</sub>	330	(A1)	Yes	No	-
S <sub>1</sub>	366	(A1, H2)	Yes	No	-
		(A1, H3)	Yes	No	-
S <sub>2</sub>	398	(A1, A2.1, A3)	Yes	Yes	Yes

## Example 24

Again using the case where all bidders' headline bids are for six B lots and all exemption lot categories, we assume that when bidders introduce additional bids:

- all bidders offer to reduce demand for B lots somewhat with bid amounts between reserve and round prices, Bidder 1 and Bidder 2 bidding for packages that include five B lots, and Bidder 3 bidding for a package that includes three B lots; and
- only Bidder 1 and Bidder 2 offer to take on the coverage obligation in any one of the coverage area groups by bidding for all possible packages that include two exemption lot categories along with five B lots, whilst Bidder 3 still includes all three exemptions in its additional bid.

We are not able to end the third auction stage as we do not have enough exemption lots (with the current bids it would only be possible to assign the coverage obligation in two of the three area groups).



Bids (round price for B lots is DKK 26 million, and for C1, C2 and C3 lots is DKK 10 million)

Bid id	Bidder	Headline/additional	B	C1	C2	C3	Amount (DKK million)
H1	Bidder1	Headline	6	1	1	1	186
A1.1	Bidder1	Additional	5	1	1	1	145
A1.2	Bidder1	Additional	5	1	1	0	140
A1.3	Bidder1	Additional	5	1	0	1	140
A1.4	Bidder1	Additional	5	0	1	1	140
H2	Bidder2	Headline	6	1	1	1	186
A2.1	Bidder2	Additional	5	1	1	1	145
A2.2	Bidder2	Additional	5	1	1	0	140
A2.3	Bidder2	Additional	5	1	0	1	140
A2.4	Bidder2	Additional	5	0	1	1	140
H3	Bidder3	Headline	6	1	1	1	186
A3	Bidder3	Additional	3	1	1	1	100

### Bid evaluation

Scenario	Highest value (DKK million)	Highest-value feasible bid combinations	Exemption-compatible?	Possible winning bid combination?	Includes a bid from each bidder?
S <sub>0</sub>	325	No bids	Yes	Yes	No
S <sub>1</sub>	361	(H1)	No	-	-
		(H2)	No	-	-
		(H3)	No	-	-
S <sub>2</sub>	397	(H1, H2)	No	-	-
		(H1, H3)	No	-	-
		(H2, H3)	No	-	-

### Price increments

Omitted bidders: Bidder 1, Bidder 2 and Bidder 3.

Omitted bidders	Lot categories identified as requiring a price increment (i.e. bidder still omitted when using hypothetical bid only including lots in this category)					All exemption lots included	All lots included
	B	C1	C2	C3	All exemption lots included		
Bidder 2	No	Yes	Yes	Yes	NA	NA	
Bidder 3	No	Yes	Yes	Yes	NA	NA	

For the next round we need to increase the price for all exemption lot categories but not for B lots.

## 5 Examples with different numbers of bidders

Until now we have seen examples with three bidders, which makes it possible to assign one coverage area group to each bidder. In this section we provide

examples with two and four bidders. The main difference relative to prior examples is that when there are exemptions available, then the number of scenarios for the supply of exemptions changes – specifically, we always have as many scenarios as bidders.

### Example 25

If there are only two bidders it is not possible to assign three A lots, so even if all bidders were to take an A lot it would be necessary to determine which one of them will take the coverage obligation in a second coverage area group.

Specifically, suppose that two bidders take one A lot each; therefore, there are nine B lots and one exemption lot category (say C1) available in the third auction stage. Given the spectrum cap, each bidder can bid for at most four B lots, so at least one B lot will remain unsold.

Given that there are only two bidders, at most one exemption lot can be assigned. Therefore we only need to consider two cases  $S_0$  and  $S_1$ .

Suppose that both bidders start making a headline bid for a package of four B lots and C1 and no additional bids. In the second round the price of all lots will increase. If both bidders keep making headline bids for the same package of four B lots and C1 and make no additional bids, then only the price for C1 will continue to increase in the following rounds.

Suppose that when prices reach DKK 26 million for B lots and DKK 10 million for C1, both bidders continue to submit a headline bid for the package of four B lots and C1, but Bidder 1 also makes an additional bid four B lots without C1 at reserve. Bidder 2 does not make additional bids.

The third auction stage can now close, with Bidder 1 winning with its additional bid and Bidder 2 with its headline bid. Note that Bidder 1 was able to compete for C1 without this meaning that it would have to pay above reserve once it decides to give up on this lot, so the only effect of the bidding process is to determine the premium to be paid for adding C1 to a package.

Bids (round price for B lots is DKK 26 million, and for C1 is DKK 10 million)

Bid id	Bidder	Headline/additional	B	C1	Amount (DKK million)
H1	Bidder1	Headline	4	1	114
A1	Bidder1	Additional	4	0	100
H2	Bidder2	Headline	4	1	114

### Bid evaluation

Scenario	Highest value (DKK million)	Highest-value feasible bid combinations	Exemption-compatible?	Possible winning bid combination?	Includes a bid from each bidder?
$S_0$	225	No bids	Yes	No	-
		(A1)	Yes	No	-
$S_1$	239	(H1)	No	-	-
		(H2)	No	-	-
		(A1, H2)	Yes	Yes	Yes

## Example 26

In this example we again assume that there are only two bidders. However, none of them take an A lot. Therefore in the third auction stage there are thirteen B lots available and all three exemption lot categories are included. Given the spectrum cap, each bidder can bid for at most six B lots, so at least one B lot will remain unsold.

As before, we only need to consider two cases  $S_0$  and  $S_1$ .

Suppose that both bidders start making a headline bid for a package of six B lots and exemptions in all categories. Suppose that in the second round prices are DKK 26 million for B lots and DKK 1 million for all exemption lot categories. Both bidders continue to submit headline bids for the package of six B lots and exemptions in all categories, but they also make additional bids for all the packages that include six B lots and two exemption lot categories at round prices, effectively offering to take the coverage obligation in any one of the coverage area groups provided that they get exemptions for the remaining two groups.

The third auction stage cannot close, as on the basis of the bids received it is not possible to assign the coverage obligation in all coverage area groups (each bidder has only bid for packages that include an exemption in at least two exemption lot categories, but we can only assign one exemption lot for each category because there are only two bidders).

A further round is needed. When we determine the need for price increments, we do not identify the need to increase prices for any of the lot categories *individually*. Although the conflict between bidders is in relation to exemptions, it is always possible to accept a hypothetical headline bid that includes only one exemption, as such a bid would fit with the additional bid submitted by the other bidder that does not include an exemption in the corresponding region. The conflict thus can only be identified when we assess whether a bidder would still be omitted if we look at the hypothetical bid that includes the headline bid demand for all exemption lot categories, but not for B lots.

Bids (round price for B lots is DKK 26 million, and for C1, C2 and C3 lots is DKK 1 million)

Bid id	Bidder	Headline/additional	B	C1	C2	C3	Amount (DKK million)
H1	Bidder1	Headline	6	1	1	1	159
A1.1	Bidder1	Additional	6	1	1	0	158
A1.2	Bidder1	Additional	6	1	0	1	158
A1.3	Bidder1	Additional	6	0	1	1	158
H2	Bidder2	Headline	6	1	1	1	159
A2.1	Bidder2	Additional	6	1	1	0	158
A2.2	Bidder2	Additional	6	1	0	1	158
A2.3	Bidder2	Additional	6	0	1	1	158

### Bid evaluation

Scenario	Highest value (DKK million)	Highest-value feasible bid combinations	Exemption- compatible?	Possible winning bid combination?	Includes a bid from each bidder?
S <sub>0</sub>	325	No bids	Yes	Yes	-
S <sub>1</sub>	334	(H1)	No	-	-
		(H2)	No	-	-

### Price increments

Omitted bidders: Bidder 1 and Bidder 2.

Omitted bidders	Lot categories identified as requiring a price increment (i.e. bidder still omitted when using hypothetical bid only including lots in this category)					All exemption lots included	All lots included
	B	C1	C2	C3	All exemption lots included		
Bidder 2	No	No	No	No	Yes	NA	
Bidder 3	No	No	No	No	Yes	NA	

For the next round we need to increase the price for all exemption lot categories but not for B lots.

## Example 27

In our last example we consider a situation with four bidders. In this case, if not all three A lots have been assigned and there are exemption lots offered in the third auction stage, we need to consider four scenarios, as we could assign up to three exemption lots in each category if all four bidders win B lots.

Suppose that two A lots have been assigned, so that in the third auction stage there are nine B lots and one exemption lot category (say C1) available. Given the spectrum cap, two bidders (those who have been assigned A lots, say Bidder 1 and Bidder 2) can bid for up to four B lots, and the other two can bid for up to six B lots.

Suppose that round prices have reached DKK 26 million for B lots and DKK 10 million for C1, and that we have the bids shown below. Bidder 1 and Bidder 2 offer some flexibility to take the coverage obligation in the remaining coverage area group, whilst the other two bidders always include C1 in their bids. All bidders also submit additional bids with fewer B lots than in their headline bids.

In this example, a further round is needed, in which only the price of B lots needs to increase.

Bids (round price for B lots is DKK 26 million, and for C1 lots is DKK 10 million)

Bid id	Bidder	Headline/additional	B	C1	Amount (DKK million)
H1	Bidder1	Headline	4	1	114
A1.1	Bidder1	Additional	4	0	100
A1.2	Bidder1	Additional	2	1	60
A1.3	Bidder1	Additional	2	0	50
H2	Bidder2	Headline	4	1	114
A2.1	Bidder2	Additional	4	0	104
A2.2	Bidder2	Additional	3	1	80
A2.3	Bidder2	Additional	3	0	75
H3	Bidder3	Headline	6	1	166
A3	Bidder3	Additional	3	1	80
H4	Bidder4	Headline	6	1	166
A4	Bidder4	Additional	3	1	75

### Bid evaluation

Scenario	Highest value (DKK million)	Highest-value feasible bid combinations	Exemption-compatible?	Possible winning bid combination?	Includes a bid from each bidder?
S <sub>0</sub>	229	(A2.1)	Yes	No	-
		(A1.1, A2.1)	Yes	No	-
		(A1.3, A2.1)	Yes	No	-
S <sub>1</sub>	243	(H1, A2.1)	Yes	Yes	No
S <sub>2</sub>	253	(H1, H2)	No	-	-
S <sub>3</sub>	254	(A1.2, H2, A3)	No	-	-

### Price increments

Omitted bidders: Bidder 3 and Bidder 4.

Omitted bidders	Lot categories identified as requiring a price increment (i.e. bidder still omitted when using hypothetical bid only including lots in this category)					All exemption lots included	All lots included
	B	C1	C2	C3			
Bidder 3	Yes	No	No	No	NA	NA	
Bidder 4	Yes	No	No	No	NA	NA	

For the next round we need to increase the price for B lots but not for exemption lots.